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Multiple atomic wave interferometry with standing-waves of light

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Abstract. Fringe shapes in a multiple-beam de Broglie-wave interferometer based on the atomic Kapitza-Dirac effect are studied. An all-optical implementation of such a device is proposed. A realization in the time-domain, using Bose-Einstein condensates released from a trap, seems viable within the present state of the art.

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During the last decade, matter-wave interferometry has been successfully extended to the domain of atoms and molecules [1]. Many different types of two-beam atom interferometers have already been demonstrated. High contrast *multiple*-beam interference fringes have been observed by Weitz *et al.* [2]. The multiple-beam splitter used in that experiment is state-selective and makes use of the particular Zeeman-level structure of cesium.

Multiple-beam interferometers in which the splitting/merging is produced with standing-waves of far-off detuned laser light in the Kapitza-Dirac diffraction regime are the subject of the present paper. Not being state-selective ("de Broglie-wave interferometry"), such an approach has the advantage of obtaining high-contrast fringes with any optically accessible atomic species and even molecules [3].

The paper consists of two parts. In a first section we study the expected fringe profiles in interferometers that use the atomic Kapitza-Dirac effect to multiply split and merge matter-wavefronts. Some particularly useful configurations are identified. A proposal for such an interferometer is made in the second part of the paper. It has the merit of employing only well established atom-optical components based on induced dipole forces. A drawback is that it could be difficult to implement using thermal beam machines. In the time domain, using ultracold atoms released from a trap, the proposal seems to be realistic. Still, a thorough numerical analysis will be required to complement the rather crude treatment given in this paper [4].

1 Multiple-beam interferometry using the atomic Kapitza-Dirac effect

Under suitable conditions, standing-waves of light can act as sinusoidal phase gratings for atoms. The far-field diffraction pattern produced by this "atomic Kapitza-Dirac (KD) effect" was first observed in reference [5]. In a three-grating Mach-Zehnder atom interferometer the KD effect has been used for the three purposes of splitting, reflecting, and merging atomic de Broglie wavefronts [6]. Since not only two, but actually a series of diffraction orders are populated by these gratings, only a fraction of the incoming atomic flux contributes to the observed (sinusoidal) fringe pattern. The efficiency may be improved either by trying to reduce the loss by "blazing" the grating, so that mainly the two relevant diffraction orders are populated [7], or by virtue of necessity, actually using the multiple-splitting property of the grating.

Here we will study the second approach. Quite generally, multiple beam interferometry is expected to lead to higher contrast fringes. Unlike the two-beam case, in which the position (and possibly the amplitude) but not the shape of the fringes is altered by external influences, in the case of a larger number of interfering beams, the fringe profile itself will be context-dependent. We will exemplarily concentrate on the fringes that arise when the distance between the splitting and the merging grating is changed.

When a standing-wave of light acts as a phase grating for atoms [8], its transmission function $T(x)$ is proportional to

$$\exp\left[\mathrm{i}\frac{\omega^2 t}{\delta}\sin^2(kx)\right], \tag{1}$$

where ω is the Rabi frequency, t the atom-light interaction time, δ the detuning and $k = 2\pi/\lambda$ the wavenumber of the light. The numerical value of the "Raman-Nath parameter" $c \equiv \omega^2 t/2\delta$ of the grating is typically in the order of a few units. We will assume that two such (identical) gratings are placed at the positions $z = -d/2$ and $z = +d/2$. An atomic matter-wavefront (de Broglie wavelength λ_{dB}) moving in the positive z-direction at a velocity v_z will be

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split by the grating at $z = -d/2$ and merged by the grating at $z = +d/2$, if we find a means to specularly reflect the various diffraction orders at $z = 0$ using an appropriate atom-optical element. The resulting interferometer would then be symmetrical. Along the y-axis the system is assumed to be homogeneous, so that the resulting diffraction problem is essentially two-dimensional.

It is important to note that, when talking about "multiple" beams, we are implying them to be spatially separated. This means that the gratings are assumed to be used in the atomic far-field diffraction regime. If a denotes the entrance aperture (the actual width over which the grating is illuminated by the atomic wavefront), the far-field condition reads $d \gg a^2/\lambda_{\mathrm{dB}}$. For the sake of readability, in this Section we will omit any finite-aperture terms [9].

The diffraction amplitudes produced by a light grating are obtained by Fourier-decomposing its transmission function T,

$$T(x) = \sum_{j=-\infty}^{\infty} J_j(c)\mathrm{e}^{\mathrm{i}j(\pi/2+2kx)} . \quad (2)$$

Here $J_j(\,)$ denotes a Bessel function. If the atomic wavefront $\psi(x)$ is initially plane and constant (normalized to unity, $\psi(x) = 1$), immediately after traversing the first grating it will be corrugated according to $\psi(x) = T(x)$. Subsequent free propagation for a time $\tau_- = (d/2)/v_z$ adds a kinetic phase factor proportional to j^2 and τ_-,

$$\psi(x) = \sum_{j=-\infty}^{\infty} J_j(c)\mathrm{e}^{\mathrm{i}j(\pi/2+2kx)} \exp\left[-\mathrm{i}j^2 \frac{(2\hbar k)^2}{2M}\frac{\tau_-}{\hbar}\right] . \quad (3)$$

Here M is the atomic mass. The mirror at $z = 0$ inverts the propagation direction of the diffracted beamlets and therefore amounts to replacing x by $-x$ in equation (3). In addition, it may introduce a j-dependent phase shift $\exp(\mathrm{i}\phi_j)$. In the next section we will consider schemes that may allow experimental implementations of such an atomic multiple mirror. Since we are assuming a symmetric interferometrical setup, we expect, to lowest order and up to a constant and therefore irrelevant phase, $\phi_j \propto j^2$. This term has the form of the kinetic phase factor, and we will absorb it in our notation for simplicity. We will take into account that, in general, the mirror will only allow to reflect the diffraction orders which are closest to the beam axis. Since only these will contribute to the interference pattern, we restrict the summation over j to the reflected orders, say, from $-J$ to J [10], and freely propagate anew for a time $\tau_+ \simeq \tau_-$, eventually obtaining

$$\psi(x) = \sum_{j=-J}^{J} J_j(c)\mathrm{e}^{\mathrm{i}j(\pi/2-2kx)} \exp\left[-\mathrm{i}j^2 \frac{(2\hbar k)^2}{2M}\frac{\tau_-+\tau_+}{\hbar}\right] . \quad (4)$$

From a ray-optical point of view, the reflecting device at $z = 0$ would be able to refocus the various incident beams only for the particular grating-mirror distance $v_z\tau = d/2$ for which it was initially devised. However, this statement holds true only up to the limits imposed by the actual, wave-optical nature of matter. In the case of atom-optical light gratings, a representative length in the Fresnel (near-field) diffraction region is the Talbot-Hiedemann self-imaging distance $D = 2(\lambda/2)^2/\lambda_{\mathrm{dB}}$ [11], which is typically in the order of a few millimeters. In fact, inasmuch as an accurate image of the grating is produced at this distance, displacing a grating by D will not essentially alter the far-field diffraction behavior. To be on the safe side we will assume that $\tau_- \simeq \tau \simeq \tau_+$ and $v_z|\tau_+ - \tau_-| \ll D$, so that the interferometer is effectively operated in the shadow-region of the gratings [11]. The second light grating then multiplies equation (4) with $T(x)$. In this symmetric interferometer, the final output into order $j = 0$ is the signal of interest. The intensity I diffracted into the zeroth order is given by

$$I = \left|\sum_{j=-J}^{J} J_j^2(c)(-1)^j \exp\left[-\mathrm{i}j^2 \frac{(2\hbar k)^2}{2M}\frac{\tau_-+\tau_+}{\hbar}\right]\right|^2$$
$$\equiv \left|\sum_{j=-J}^{J} J_j^2\,(c)\,(-1)^j \mathrm{e}^{-\mathrm{i}j^2\alpha}\right|^2 . \quad (5)$$

We now proceed to identify the parameters c for which the shape of the fringes $I(\alpha) = I(\alpha+2\pi)$ may particularly advantage interferometry.

J = 1: three-beam atom interferometer

In this case we have $I(\alpha) = J_0^4 + 4J_1^4 - 4J_0^2J_1^2\cos\alpha$. The best contrast is obtained when the condition $J_0^2 = 2J_1^2$ is fulfilled, leading to $I(\alpha) = [4J_1^2\sin(\alpha/2)]^2$. The smallest c satisfying this condition is $c \simeq 1.161$, leading to sinusoidal fringes with a contrast $[4J_1^2(c)]^2$ of about 91% (see Fig 1), much better than the maximal contrast achievable in a three-grating Mach-Zehnder interferometer. Zero contrast is obtained if we choose c such that $J_0(c) = 0$ (dashed line in Fig. 2). This is physically obvious due to the symmetry of the resulting two-beam interferometer.

J = 2: five-beam atom interferometer

In this case we obtain

$$I(\alpha) = J_0^4 + 4J_1^4 + 4J_2^4 - 4J_0^2J_1^2\cos\alpha - 8J_1^2J_2^2\cos(3\alpha) + 4J_0^2J_2^2\cos(4\alpha) . \quad (6)$$

If we choose the same condition $J_0^2 = 2J_1^2$ as in the $J = 1$ case, we obtain the solid curve plotted in Figure 1. Its maximum is substantially sharper than in the three-beam situation. The maximum is about $I(\pi) \simeq 0.996$, implying that most of the incoming atomic flux can be captured in this configuration. In fact, using this parameter, the addition of two more reflected beams ($J = 3$) does not

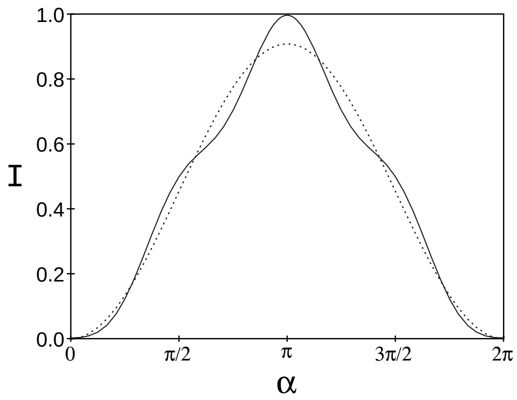

Fig. 1. One period of the three-beam (dotted line) and five-beam (continuous line) interferometer fringe signal I for a Raman-Nath parameter $c = 1.161$.

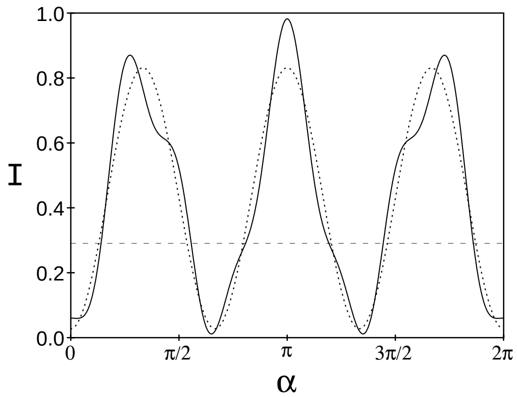

Fig. 2. One period of the four-beam (dotted line) and six-beam (continuous line) interferometer fringe signal I for a Raman-Nath parameter $c = 2.405$. No contrast would be observed in a corresponding two-beam interferometer (dashed line).

appreciably change the fringe shape. The contrast raises to 99.99%.

Another interesting possibility is choosing c such that $J_0(c) = 0$. Then we are effectively dealing with a *four*-beam atom interferometer that produces three sinusoidal fringes, $I(\alpha) = 4[J_1^4 + J_2^4 - 2J_1^2 J_2^2 \cos(3\alpha)]$, per period, thus increasing the sensitivity of the interferometer by a factor of three. Since the lowest zero of J_0, $c \simeq 2.405$, is quite close to the value $c = 2.630$, for which $J_1^2(c) = J_2^2(c)$ is satisfied, the contrast is quite high (about 83%). The (dotted) curve representing the fringe shape under these circumstances is shown in Figure 2.

J = 3: seven-beam atom interferometer

As we already commented, two more reflected beams do not substantially change the performance of the interferometer in the $c = 1.161$ configuration. However, if $J_0(c) = 0$ is fulfilled (meaning that we are actually dealing with a six-beam interferometer), the fringes become non-sinusoidal and the central maximum at $\alpha = \pi$, in particular, becomes strongly peaked, with $I(\pi) \simeq 0.98$. At the minima, less than 2% of the incoming atomic flux is diffracted into the zeroth order. The shape of these fringes is shown in Figure 2 (continuous line) .

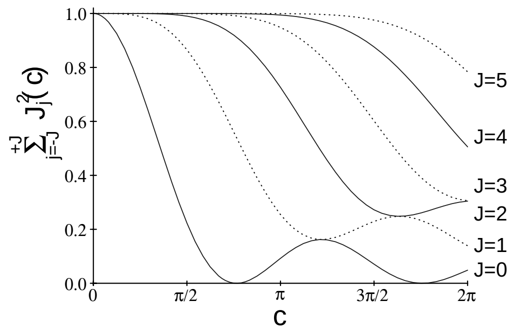

Fig. 3. Fraction of the incoming flux diffracted into the orders $-J$ to J by a sinusoidal phase grating, as a function of its Raman-Nath parameter c.

J > 3: (2J + 1)-beam interferometer

The fraction, $\sum_{j=-J}^{J} J_j^2(c)$, of the incoming atomic flux that a light grating diffracts into the orders $-J...J$ is shown in Figure 3 as a function of the Raman-Nath parameter c. Evidently, the larger c, the more diffracted beams have to be reflected by the mirror at $z = 0$ in order to collect most of the flux and achieve high contrast fringes. We see that, up to values $c \simeq \pi$, no substantial increase in contrast will be obtained by reflecting more than 7 beams. Since such c values are rather typical experimentally, we will not go beyond the $J = 3$ interferometer in this preliminary, investigative analysis.

2 On a possible implementation using an array of parallel atom-mirrors

In principle, a convergent atom-lens of focal length $f = d/4$, placed at $z = 0$, could be used to redirect the diffracted beams so as to become remixable. One of the motivations for studying a KD multiple-beam interferometer was the observation that dispersive interactions can be used to coherently manipulate a broad class of atomic species. For this reason, we will only consider all-optical approaches to beam-reflection. A nearly parabolic, convergent lens for atoms made of far-off detuned laser light was first demonstrated by Sleator *et al.* [12]. It employed one intensity period of a standing-wave of light produced by grazing incidence reflection on a glass surface. The aperture a was about 25 μm and the focal length f about 30 cm in that experiment using metastable helium. In a KD three-grating Mach-Zehnder experiment using metastable argon [6] the distance $d/2$ between the splitting (merging) and the reflecting grating was 25 cm and the spacing between two consecutive diffraction orders at $z = 0$ was about 8 μm. This indicates that at least the involved orders of magnitude would be appropriate for such an approach. It must be noted, however, that the far-field condition, using the parameters from [6], reads $z \gg 2$ m, meaning that the interferometer described in that reference does neither really operate in the far- nor in the near-field. A proper description of a multiple-beam interferometer using a parabolic light lens and two KD gratings operated in the intermediate field requires a detailed numerical analysis that is clearly beyond the scope of this paper.

From now on we will assume that the far-field condition is well satisfied, *i.e.*, that the number $m \equiv d\lambda_{\mathrm{dB}}/a^2$ is chosen large enough. For distances $\tilde{z}$ (behind a grating) corresponding to the far-field regime, both the width $a(\tilde{z}) \geq a$ of the diffracted beams, $a(\tilde{z}) \propto \tilde{z}\lambda_{\mathrm{dB}}/a$, and their mutual separation $s(\tilde{z})$ along the x-axis, $s(\tilde{z}) = \tilde{z}\lambda_{\mathrm{dB}}/(\lambda/2)$, grow linearly with $\tilde{z}$. We will require the various beams to be spatially well separated, meaning that the geometrical ratio $n \equiv s(z=0)/a(z=0) = a/(\lambda/2)$ at the symmetry plane of the interferometer should greatly exceed one. In order to avoid "coloured" interference fringes, a good monochromatization of the molecular beam is mandatory. In particular, this guarantees that the diffraction spots and their positions at $z = 0$ are well defined. Their mutual separation will then be given by $s = m^2n(\lambda/2)$. If the light lens at $z = 0$ is produced by grazing incidence reflection on a glass surface, the resulting intensity period must exceed the minimum value $\lambda/2$ by a factor of at least m^2n (a factor of about 100 has been reported in Ref. [12]). Since we are intending to reflect *several* diffraction orders, the aperture of the light lens would actually have to be even larger. Although there is no in-principle limit to larger intensity periods, there do exist experimental limitations for such an extrapolation. Other approaches could be considered as well. For instance, the optical potential close to the intensity maximum of a tightly focused laser beam could be used as an atom lens with the required aperture [13]. For the present discussion, however, we prefer a more conservative approach based on methods that have already been successfully demonstrated in atom-optics experiments.

Since we are assuming a situation in which the spacing s between the various diffraction orders is well defined, constant and known, it would be enough to place an array of parallel mirrors with mutual spacing s at $z = 0$, instead of a single, wide lens. Nature provides us candidates for highly parallel and equally spaced structures in the form of standing-waves of light. The idea is sketched in Figure 4. We see that, in the above picture, it essentially amounts to dedicating one lens to every diffracted beam. If $\Lambda/2$ denominates the spatial intensity period of the broadened standing-wave and $K = 2\pi/\Lambda$ the corresponding wave number, within the same approximations that led to expression (1) the resulting "optical" potential V will be given by

$$V = \frac{\hbar\Omega^2}{\Delta}\sin^2(Kx)\,. \tag{7}$$

Again, Ω and Δ represent the interaction Rabi frequency and the detuning, respectively. In particular, the validity

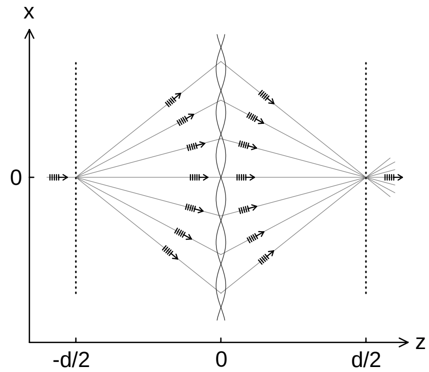

Fig. 4. Scheme of a multiple-beam Kapitza-Dirac far-field atom interferometer, in which the multiple-beam reflection at $z = 0$ is accomplished with the help of an expanded standing-wave of near-resonant light. The beam splitter (merger) at $z = -d/2$ ($z = d/2$) is indicated with a dotted line.

of the semiclassical ("Raman-Nath") approximation that leads to the transmission function (1) and which basically amounts to neglecting the kinetic energy term in the interaction Hamiltonian [11] will also be assumed here and easily justified *a posteriori*. Under such circumstances, a mechanistical point of view is in order [14]. During the interaction with the expanded standing-wave the atoms feel a force which is the negative derivative, $-\mathrm{d}V/\mathrm{d}x$, of the optical potential. If the difference

$$\frac{\Lambda}{2} - s \ll \frac{\Lambda}{8} \tag{8}$$

and for large enough n, we can approximate the force F_j acting on an atom of the jth order diffracted beam by

$$F_j = -\frac{2\hbar\Omega^2K^2}{\Delta}\left[\frac{\Lambda}{2} - s\right]j\,. \tag{9}$$

The angles between the various diffraction orders are typically smaller than a milliradian and have been highly exaggerated in Figure 4 for illustrative reasons only. As a consequence, the interaction time T of the atoms with the standing-wave of light at $z = 0$ does not depend on j essentially.

Physically speaking, the "Raman-Nath" regime corresponds to the situation in which the atom does not have enough time to substantially move along the x-axis during the interaction. The lateral momentum transfer on the jth beamlet is then essentially given by F_jT. Since each diffraction order corresponds to a photon momentum transfer of $2\hbar k$, the reflection condition reads $F_jT = -2(2\hbar k)j$, or

$$\frac{\Lambda}{2} - s = \frac{k}{K^2}\frac{2\Delta}{\Omega^2T}\,. \tag{10}$$

Using this relation and introducing the Raman-Nath parameter $C \equiv \Omega^2T/2\Delta$, the inequality (8) becomes

$$\frac{\Lambda}{\lambda} \ll \frac{\pi}{4}C \tag{11}$$

and makes evident that the reduced intensity gradient due to $\Lambda/\lambda \gg 1$ must be compensated by increasing the magnitude of C correspondingly. The previously defined maximum number J of reflected beams may be written

$$J \simeq \mathrm{int}\left(\frac{\Lambda/8}{\Lambda/2 - s}\right). \tag{12}$$

By choosing

$$C = c\left(\frac{\Lambda}{\lambda}\right) J \frac{t}{T}, \tag{13}$$

the inequality (11) may be reasonably well satisfied, especially if T is chosen sufficiently shorter than t. Such a choice also guarantees that the Raman-Nath condition for the reflection grating at $z = 0$, $\pi ChT \ll M\Lambda^2$, is well satisfied if the analogous inequality, $\pi cht \ll M\lambda^2$, holds for the diffraction gratings at $z = \pm d/2$.

In order for the involved atom-optical elements to act as pure phase objects, satisfying the Raman-Nath condition is indeed a necessary requirement. In experiments performed with thermal atoms, this implies that the interaction time has to be shortened as much as possible by focusing the laser beams close to the diffraction limit. This may prove fatal both for the gratings and for the mirror array in our scheme. Due to the strong focusing, the entrance aperture a of the gratings is basically defined by the lateral width of the laser focus, which typically comprises a rather small number of standing-wave periods only [5, 6]. As a consequence, the ratio $a/(\lambda/2)$ may not be large enough to guarantee a clean separation of the diffraction orders at $z = 0$. In addition, there remains the problem of the physical length of the interferometer due to the requirement $d \gg a^2/(\lambda_{\mathrm{dB}})$. A strong focusing is also hardly compatible with the required parallelity of the constant intensity planes in the reflection (middle) grating. As far as the focal length of the reflection grating depends on the laser intensity, its stability needs to be carefully controlled during the interference experiment. Extremely stable sources have been recently used for trapping atoms in a laser focus [13].

An implementation in the time domain, using cold atoms released from a trap and interacting with properly timed laser pulses, would overcome these problems. The Raman-Nath condition can then be fulfilled by choosing short enough light pulses without compromising the parallelity of the light intensity ripples. In particular, the temporally spaced light gratings would be mutually parallel without further ado and the fulfillment of a far-field condition does not imply prohibitive interferometer lengths. In addition, the required high intensity and intensity stability of the laser which produces the standing-wave mirror array may be easier to achieve with a pulsed (instead of cw) light source. Recent experiments employing released Bose-Einstein condensates interacting with pulsed standing-waves [15] show that an excellent separation of the various diffraction orders can be obtained, due to the large spatial coherence of the atom source. Since the interaction time is defined by the pulse length and not by the time-of-flight through the interaction region, chromatical aberrations are highly suppressed in time-domain experiments. Taking the experimental data (in brackets) from [15] as a reference, we may choose the following parameter values. Let $m = 3$ ($m = 3.07$) and $n = 100$ ($n = 140$). This means that a de Broglie wavefront of Na atoms ($M = 23$ amu, $\lambda = 589$ nm) should be initially coherently delocalized over 50 wavelengths, *i.e.* $a \simeq 30\,\mu$m ($a \simeq 40\,\mu$m). We choose $c = 2.405$ ($c = 2.827$), corresponding to the $J_0(c) = 0$ interferometric configuration. The pulse time t is chosen as in the paper ($t = 100$ ns). The free evolution time $\tau_+ \simeq \tau- = 4.2$ ms (6.2 ms) is long enough to guarantee that we are operating in the far-field diffraction regime. The mutual spacing between the diffracted beamlets is $s = 0.256$ mm ($s = 0.378$ mm). If the interferometer mirrors are produced by a light grating with $\Lambda = 1000\lambda$, it should in principle be possible to reflect diffraction orders up to $j = \pm 2$, *i.e.* $J = 2$. Setting $T = t$, condition (11) is well fulfilled, $0.26 \ll 1$, if C is chosen according to equation (13). As was already emphasized, this implies that the intensity of the reflection grating must be correspondingly increased by a factor of roughly Λ/λ. Pulsed laser sources developed in the context of atom optics and interferometry by the Hänsch group [16] allow to deliver powers in the order of several 100 W in the form of pulses in the ~ 10 ns duration range. They would also be ideally suited for producing the time-domain mirror in the multiple interferometric scheme proposed in the present paper.

Summary

We have theoretically studied the fringe shapes produced by a symmetric multiple-beam far-field matterwave interferometer based on the atomic Kapitza-Dirac effect. A number of interesting parameter values are identified and a possible implementation is discussed. A realization in the time-domain, using Bose condensates released from a trap, seems practicable.

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